

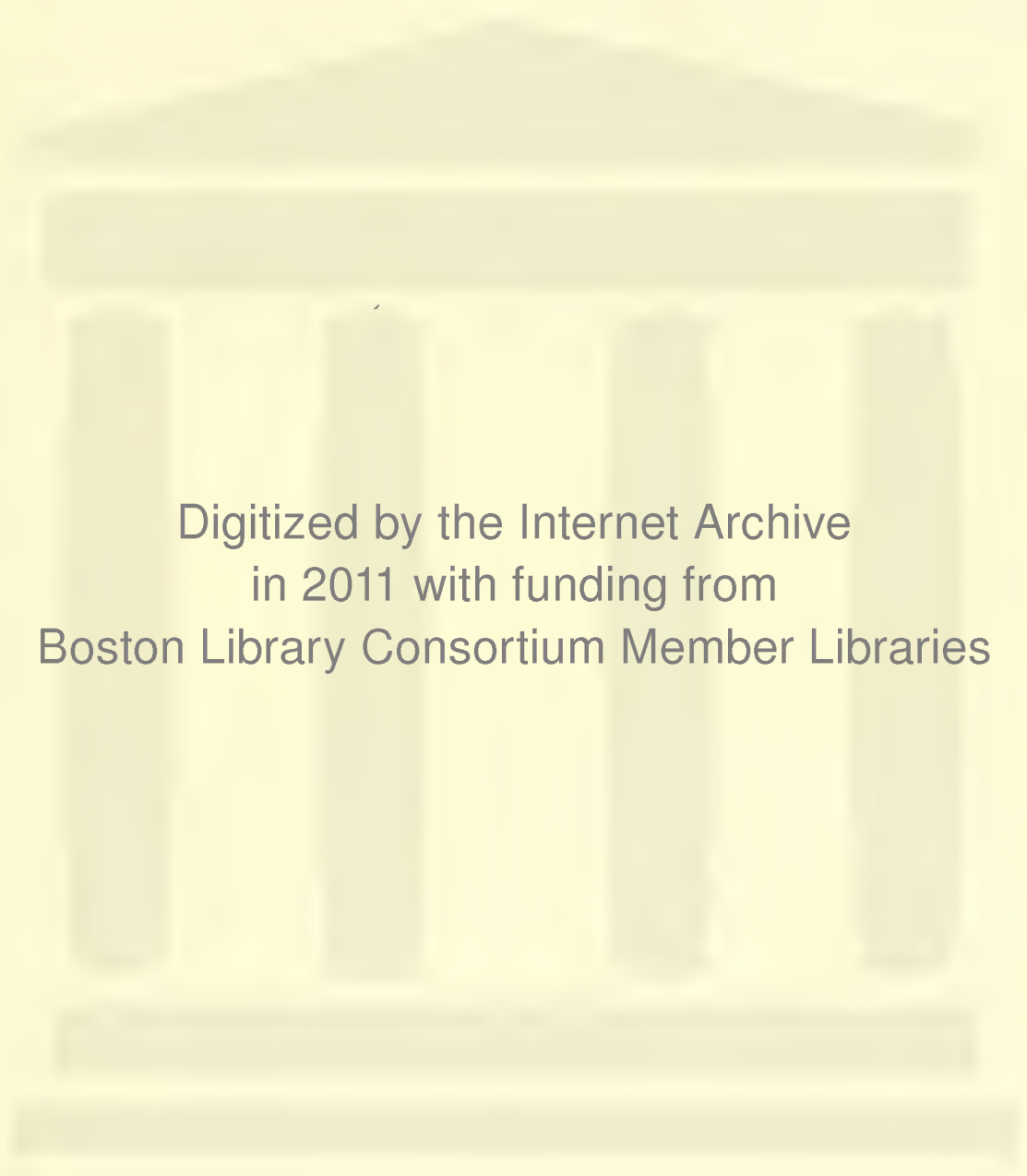
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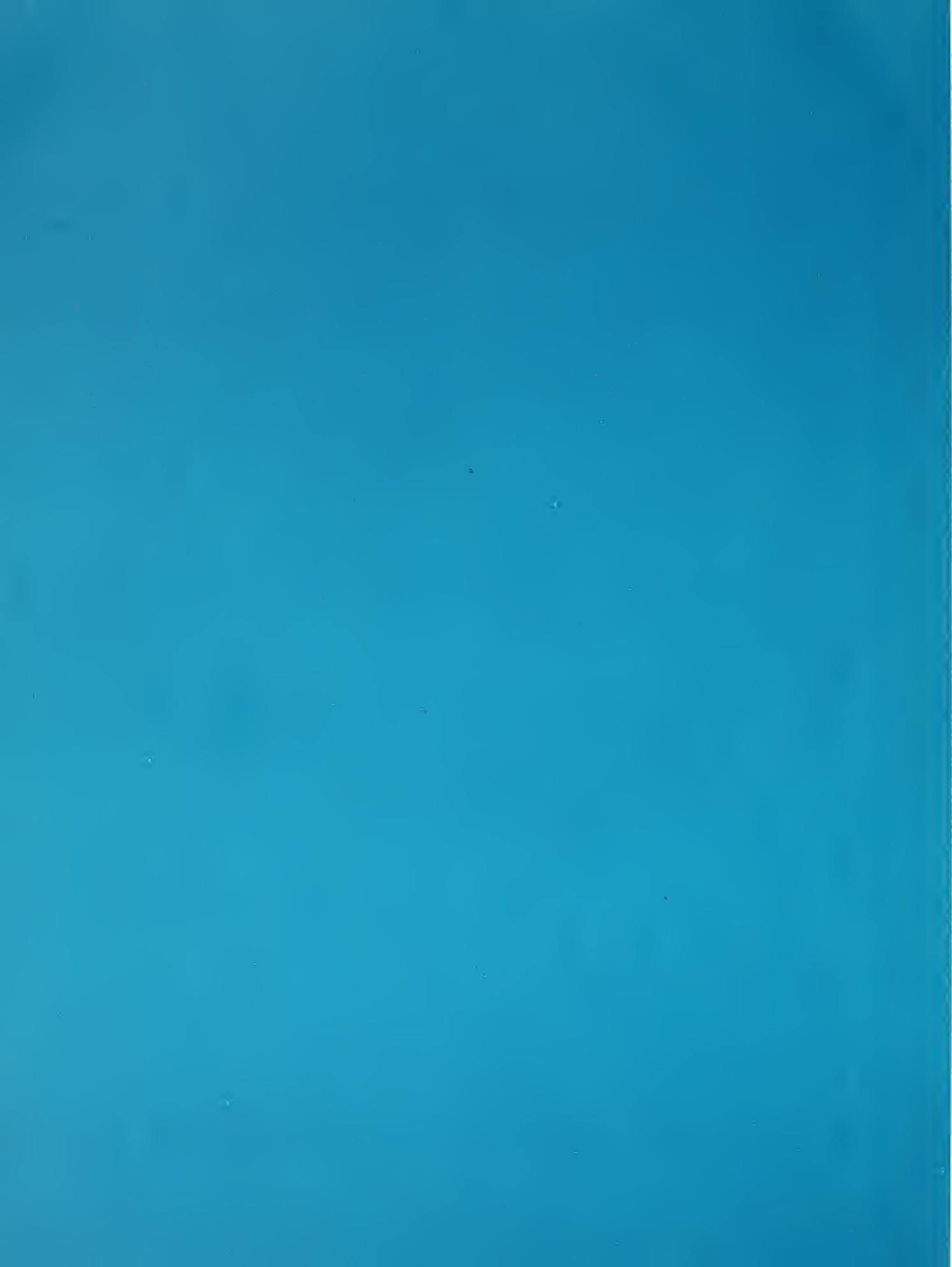
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**working paper
department
of economics**

MONEY IN SEARCH EQUILIBRIUM

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Number 297

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MONEY IN SEARCH EQUILIBRIUM

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Money in Search Equilibrium

P. Diamond*

1. Introduction

Some economists attribute fluctuations in unemployment to misperceptions of prices and wages. Others attribute such fluctuations to lags in adjustment of prices and wages (including staggered contracts). It seems to be a shared view that there would be no macroeconomic unemployment problems if prices and wages were fully flexible and correctly perceived. This paper examines a third cause for macro unemployment problems - the difficulty of coordination of trade in a many person economy. That is, once one drops the fictional Walrasian auctioneer and introduces trade frictions, one can have macro unemployment problems in an economy with correctly perceived, flexible prices and wages. This proposition is analyzed in a model where money plays a critical role in coordinating transactions.

To model the transactions role of money, it seems natural to use a continuous time model where transactions occur at discrete times.** This

*This paper and my earlier one (1982) were presented as the Fisher-Schultz lecture at the Amsterdam Econometric Society Meetings in 1981. I am grateful for useful discussion with C. Wilson, research assistance by M. Whinston, and research support from NSF.

**For an example of a discrete time macro model based on a continuous time micro model, see Akerlof (1973).

approach conforms with much of common experience and avoids the need to find an appropriate discrete time constraint on individual transactions within a period. It also seems natural to require search for all transactions, rather than closing a partial equilibrium search model with frictionless competitive markets.

This paper presents a simple general equilibrium search model where money is used for all transactions (no barter or credit). The model considers only steady state rational expectations equilibria. Individual production decisions determine the flow of goods into inventories for trade. The aggregate flow of transactions depends on the size of inventories and the stock of individuals with money trying to purchase. Individuals experience the arrival of trading opportunities as a Poisson process with parameters consistent with the aggregate flow of transactions. When a buyer and a seller meet, they negotiate a price for the sale of one unit of the indivisible consumer good. It is assumed that negotiations are always successful and that the negotiated price divides evenly the utility gain from completing the transaction. Thus, individuals produce, sell their output, and use the money to buy output from others.

For a given constant nominal money supply, there are multiple equilibria, with equilibria with a higher level of production associated with lower prices and higher real money supplies. That is, an economy with this structure of transaction costs has multiple natural rates of unemployment. This suggests that in the presence of macro shocks, macro policy should vary with the position of the economy to keep the economy from spending long periods of time in a low level equilibrium.

A further finding is that none of the internal equilibria of the economy are efficient relative to policies which could vary the incentive to produce and the

real money supply. We do not examine policies to affect those variables since this would take us out of the simple steady state equilibrium.

The paper begins with a simple overview of the model (Section 2). The trade process is presented next (Section 3), then the production process (Section 4), and then optimal individual production decisions (Section 5). The rule for price determination and analysis of equilibrium are in Section 6. Local efficiency is discussed in Section 7. The model is discussed and given an alternative interpretation in the concluding sections. Appendix A considers equilibrium and efficiency for exogenous prices.

2. Overview

Before presenting the precise specification of the model, we start with a parable which describes the basic mechanism. Consider a tropical island with many people. Individuals walk along the beach looking for palm trees. Each tree has one nut. Trees differ in height and there is a disutility to climbing. Having found a tree an individual must decide whether to climb it. There is a taboo on eating nuts one has picked oneself. Thus, having climbed a tree, an individual must engage in trade to enjoy any consumption. The assumed taboo plays the role of the advantage of specialization and trade over self-sufficiency in a modern economy. For convenience, assume that an individual never chooses to climb a second tree when he has a nut in inventory.

There is a fixed quantity of fiat money on this island. Having picked a nut, the individual looks for someone with money to whom to sell the nut. By assumption there is no barter. This represents the fact that the problem is really finding a nut to one's taste rather than any nut. For simplification there is no credit. When a seller finds a buyer, they negotiate a mutually

agreeable price. This mutually agreeable price is such that the gain to the seller from the transaction (rather than waiting for the next buyer) is equal to the gain to the buyer (rather than waiting for the next seller). Having sold his inventory, the individual goes shopping with the money received. After completing a purchase, the individual goes back to searching for short trees.

In this setting, one can consider rational expectations steady state equilibria. That is, we consider equilibria where (1) the production decision is individually optimal given the correctly perceived parameters of the production and trade processes and (2) the pricing rule is based on the correct evaluation of the gains in expected utility from carrying out transactions. We find that the economy has multiple equilibria and that these are inefficient relative to policies which can directly control production incentives and the real money supply.

3. Trade Process

We begin by describing the technology controlling the matching of buyers and sellers. This is made up of two parts (1) a determinate aggregate trade function relating the flow number of trades to the stocks of buyers and sellers and (2) Poisson processes giving the stochastic rates of transactions for each individual.

Assume there are m individuals with money trying to buy the commodity and e individuals employed in the trade process with inventories they are trying to sell. (With n individuals in aggregate, $n-e-m$ of them are searching for production opportunities.) Then, the rate of completion of transactions will be written as $f(e,m)$, which is assumed to be twice continuously differentiable and to have well behaved isoquants.

We make a number of assumptions about the trade technology. First, we assume positive marginal products provided a positive number on the other side of the market

$$f_e(e,m) > 0, f_m(e,m) > 0 \text{ when } m > 0, e > 0 \quad (1)$$

Second, we make the assumption of increasing returns to scale

$$ef_e + mf_m > f \quad (2)$$

Increasing returns to scale are very plausible at low levels of activity. They also become plausible at higher levels once one recognizes the geographic dispersion of buyers and sellers and thus the gain from increased transaction locations as the density of traders grows. In a business cycle context, with a fixed infrastructure of trading capacity, there is increased short run profitability from increased use of retail trade facilities.* Third, we assume that the returns to scale are not so large that the average rate of transactions rises with an increase in traders on that side of the market. Differentiating f/e with respect to e and f/m with respect to m , we have

$$f \geq ef_e, f \geq mf_m \quad (3)$$

To go with this description of aggregate outcomes we need a consistent description of the individual experience of the trade process. We assume that each seller experiences the arrival of buyers as a Poisson process with arrival rate b and each buyer experiences the arrival of sellers as a Poisson process with arrival rate s . Individuals view these rates as parameters beyond their control. Thus, we are ignoring search intensity, advertising, and reputation as determinants of the outcome of the search process. The results of this analysis seem robust to inclusion of these elements, since optimization over additional

*For an argument that macro problems are inherently connected with increasing returns to scale see M. Weitzman (1981).

control variables would still leave the profitability of production for trade an increasing function of potential trading partners.

For micro-macro consistency, the sum of individual experiences must equal the aggregate experience, which has been assumed to be nonstochastic. Thus we have

$$b_e = s_m = f(e, m) \quad (4)$$

That is, the arrival rate of buyers times the number of sellers equals the arrival rate of sellers times the number of buyers equals the aggregate rate of meetings. Since there are no further matching problems and the negotiation process is assumed to be instantaneous and always successful, the rate of meetings equals the rate of transactions.

4. Production

Rather than modelling production as a continuous process with varying intensity, we follow the slightly simpler course of viewing production as instantaneous, with the search for production opportunities as time consuming. Thus each individual not engaged in buying or selling has the possibility of finding a production opportunity. The arrival of production opportunities is a Poisson process with arrival rate a . Each opportunity represents one indivisible unit of output available for sale. Each opportunity involves a labor cost, c , which is an independent draw from the exogenous distribution $G(c)$. It is assumed that there is a strictly positive minimal cost of production $\underline{c}$. That is, the support of G is bounded below by $\underline{c}$. For convenience we also assume that the support is connected and unbounded and G is differentiable.

Only those not engaged in buying or selling are searching for production opportunities. If all projects costing less than c^* are taken, the rate of change in the stock of inventories satisfies

$$\dot{e} = aG(c^*)(n-e-m) - f(e,m) \quad (5)$$

That is, inventories grow from production by searchers who accept all projects costing less than c^* and shrink from sales. We will be concentrating on steady state equilibria where $\dot{e}$ is zero. We will also be considering equilibria with constant prices. Then, the number of shoppers, m , does not change since a completed transaction converts a buyer into a searcher for production opportunities and a seller into a buyer.

5. Individual Choice

We now model individual choice of c^* , the cutoff cost of production opportunities undertaken. We shall consider only steady states where b and s do not vary. With a constant price level, the sale of one unit is just sufficient to finance the purchase of one unit. Thus we do not need to pay attention to the details of money holdings, just to whether an individual is in a position to buy (i.e. has enough money for a unit of purchase), is in a position to sell (has inventory), or neither of the above. We denote the three states by e , m , and u . The disutility of a completed production opportunity is c . The utility of consumption coming from a completed purchase is denoted y . It is assumed that the individual has a positive discount rate r , lives forever, and seeks to maximize the expected present discounted value of consumption utilities less production disutilities. We denote the expected present discounted values of lifetime utility conditional on currently being employed in selling, having money to buy, and being in neither of these states as W_e , W_m , and W_u . Using a dynamic

programming framework, the rate of discount times the value of being in a position is equal to the expected gains from instantaneous utility and from a change in status. Thus we have the three value equations

$$rW_e = b(W_m - W_e) \quad (6)$$

$$rW_m = s(y + W_u - W_m) \quad (7)$$

$$rW_u = a \int_0^{c^*} (W_e - W_u - c) dG(c) \quad (8)$$

That is, those with inventory wait for buyers; those with money, for sellers; and those with neither, for production opportunities. The optimal cutoff rule is to undertake any project that costs less than the gain in expected utility from undertaking a project. That is,

$$c^* = W_e - W_u \quad (9)$$

Subtracting the equations pairwise and using (9) we have

$$rc^* = b(W_m - W_e) - a \int_0^{c^*} (c^* - c) dG \quad (10)$$

$$(r+b)(W_m - W_e) = s(y + W_u - W_m) \quad (11)$$

$$(r+s)(W_u - W_m) = a \int_0^{c^*} (c^* - c) dG - sy \quad (12)$$

Substituting from (11) and (12) in (10) we have an implicit equation for c^* :

$$(r+b)(r+s)c^* = bsy - (r+b+s) a \int_0^{c^*} (c^* - c) dG \quad (13)$$

We will refer to the solution of (13) as $c^* \left(\frac{bs}{r+b+s} \right)$ since all terms in b or s can be written in this form.* We note that $c^* \left(\frac{bs}{r+b+s} \right)$ is increasing, zero if either b

* $r(r+b+s) = (r+b)(r+s) - bs$

or s is zero, and tending to y if b and s both rise without limit.

Differentiating (13) we have

$$\begin{aligned} ((r+b)(r+s) + (r+b+s)aG(c^*)) \frac{\partial c^*}{\partial b} &= \left(\frac{s}{r+b}\right)(ry+a \int_0^{c^*} (c^*-c)dG) \\ ((r+b)(r+s) + (r+b+s)aG(c^*)) \frac{\partial c^*}{\partial s} &= \left(\frac{b}{r+s}\right)(ry+a \int_0^{c^*} (c^*-c)dG) \end{aligned} \quad (14)$$

6. Price Determination

If prices are exogenous and the real money supply fixed, equilibria are the solutions to $\dot{e} = 0$, evaluated at $c^*(\frac{bs}{r+b+s})$. If individuals negotiate the prices at which they trade, while the government controls the nominal money supply, then some of the steady states with arbitrary real money balances are not equilibria.

In this section, we will consider possible steady state equilibria when a particular pricing rule holds. After this analysis, I will discuss the motivation for the pricing rule. In Appendix A we consider equilibria with exogenous real money supplies.

When a seller and buyer complete a deal, each of them has a surplus relative to his next best alternative, which is waiting to complete a deal with the next trader to be met. The problem of price negotiation is the problem of the division of this surplus between buyer and seller. We assume that prices are chosen so that the utility gain to the buyer equals the utility gain to the seller.* That is, the gain from the change of status from seller to buyer is equated with the gain from consumption less the loss from change of status from buyer to searcher for production opportunities

$$W_m - W_e = y + W_u - W_m \quad (15)$$

*This is the Raiffa bargaining solution. See Luce and Raiffa Section 6.10.

Combining (15) with the equation for the values of different positions, (11), we see that the pricing rule implies

$$r + b = s \quad \text{or} \quad r + \frac{f}{e} = \frac{f}{m} \quad (16)$$

That is, if we are in a steady state equilibrium satisfying the equal sharing of the trading surplus, then the rate of interest plus the arrival rate of buyers equals the arrival rate of sellers. Implicitly differentiating (16), we know that m and e are positively related:

$$\left. \frac{de}{dm} \right|_{r+b=s} = \frac{\frac{f_m}{e} + \frac{f - mf_m}{2m}}{\frac{f_e}{m} + \frac{f - ef_e}{2e}} > 0 \quad (17)$$

We assume that (16) appears as in Figure 1.* That is, for $e > \underline{e}$, there is a solution to (16) and $f(\underline{e}, m)/m$ goes to r as m goes to zero. We are assuming that the arrival rate of sellers becomes very small as inventories become small, even if the number of shoppers is much smaller.

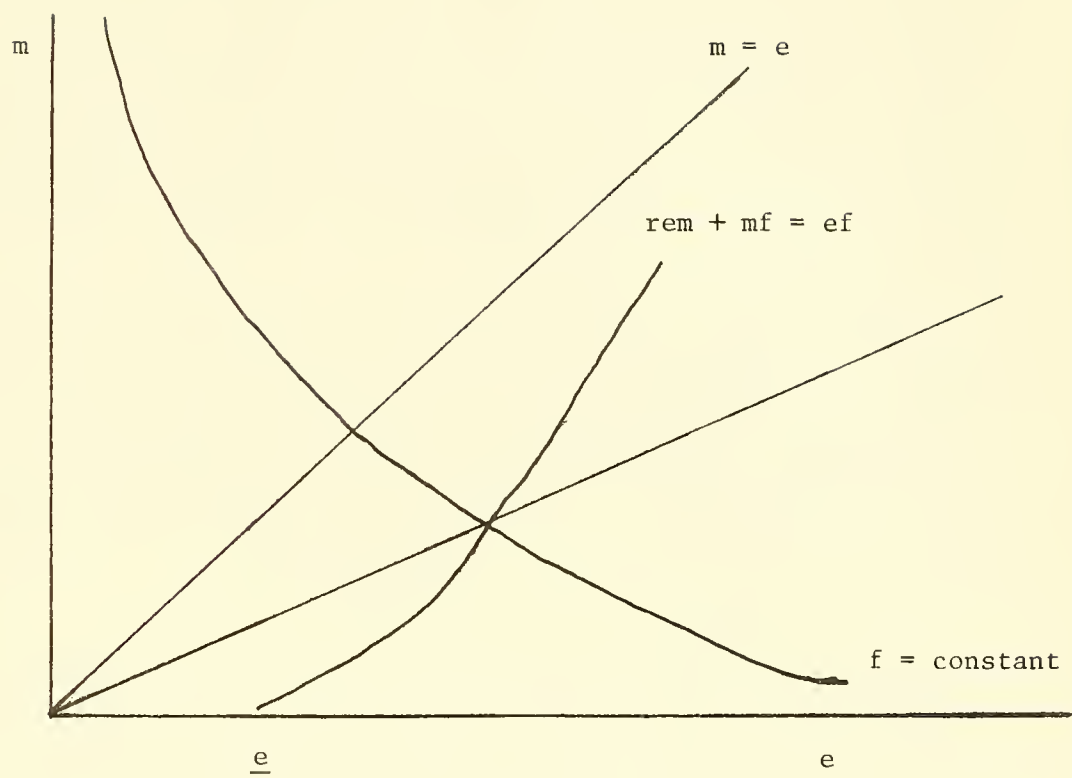
Using (16), we can examine optimal choice, c^* , and the steady state condition, $\dot{e}=0$, in a single diagram in (c^*, e) space. Substituting from (16) in (13), we have

$$c^* = \frac{\frac{f}{e} y + 2a \int_0^{c^*} cdG}{2r + \frac{f}{e} + 2aG} \quad (18)$$

Analyzing (18), using (16) to relate m to e , we see that $c^*(e)$ is an increasing function with $c^*(\underline{e}) = 0$ and an upper bound no greater than y (with a positive discount rate no one would undertake a project that costs more than its eventual gain). Analyzing $\dot{e} = 0$, using (16) to relate m to e , we see that $\dot{e} = 0$ has the shape shown in Figure 2 - vertical above $\underline{e}$ to $\underline{c}$, c^* and e positively related

*The analysis would be basically unchanged with $\underline{e}$ equal to zero provided s went to zero in the neighborhood of the origin.

Figure 1



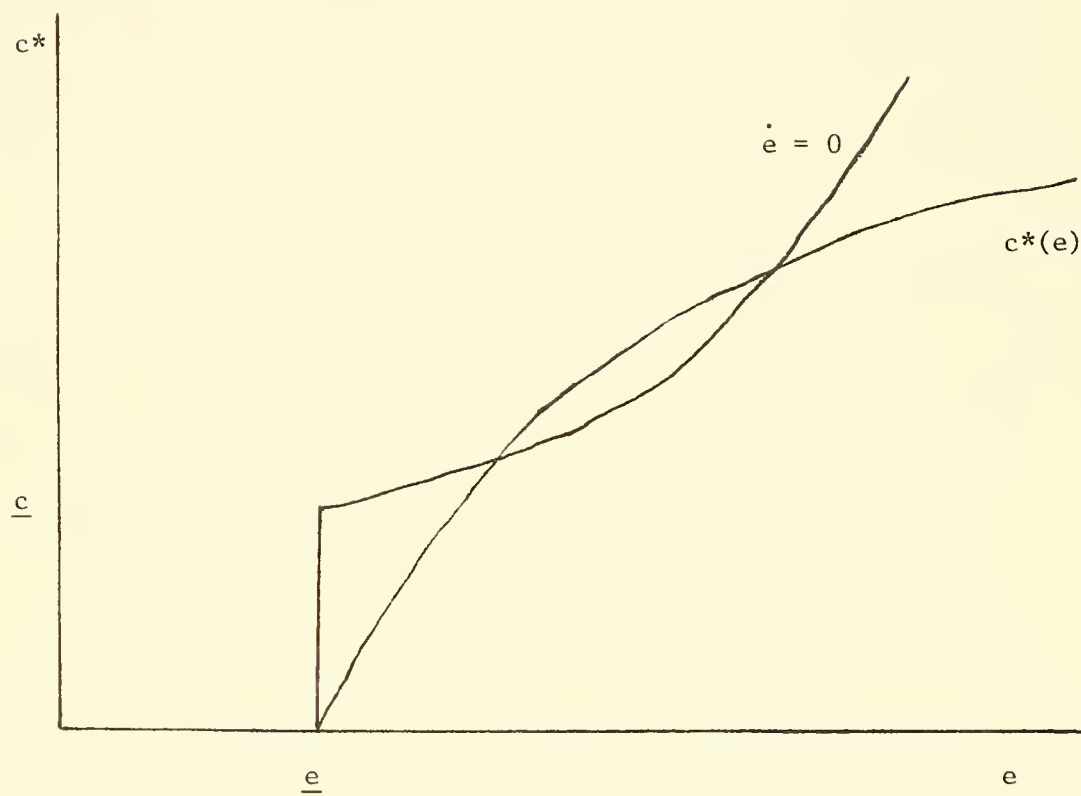
above $\underline{c}$, and a vertical asymptote at a finite level of e , representing the maximal sustainable inventories when all projects are undertaken. Combining these curves in a single diagram, Figure 2, we see the possibility of multiple equilibria. Thus when price setting satisfies the equal utility gain condition and the economy has the potential of operating at a positive level, there are multiple steady state equilibria. Those equilibria with higher willingness to produce have greater stocks of inventories and greater real money supplies (lower prices for a given nominal money supply.) There may be many more equilibria than the three shown in the diagram.

Having described equilibrium with the equal utility gain assumption, it remains to justify its use. I shall not review the discussion of its appropriateness as an equilibrium concept in bargaining theory. Rather I will describe a peculiarity of the bargaining situation in this model and mention extensions of the model which would address them.

In an equilibrium where all goods sell at the same price and all money holdings are in unit multiples of the price of a good, there must be sticky prices. The attempt to charge a higher price than the going price must fail since no buyer has more money. The attempt to bargain for a lower price is limited by the difficulty of estimating the ability to purchase of someone who has slightly less money than the going price; there are several consistent conjectures, giving different size ranges of equilibria. (Equilibria might not occur at a particular price since it remains necessary to have an adequate incentive for positive production.)

Thus, at first examination, bargaining theory appears capable of supporting many equilibria, not just those compatible with the equal utility gain assumption. To use the equal utility gain assumption without complication,

Figure 2



equilibrium should have individuals with a continuum of money holdings so that there is a unique evaluation of the consequences of having slightly less or slightly more money. With a distribution of money holdings, bargaining theory implies a distribution of trading prices. Thus, the analysis is more complicated than what we have done so far. In future work, I hope to show that as parameters vary, the equilibria in such an economy converge to the one analyzed here. That will justify the use of the equal utility gain assumption in this much simpler model, where the possibility of sticky prices comes from an artificial aspect of the model rather than the phenomenon being analyzed.

7. Local Efficiency

An increase in the willingness to produce (money held constant) makes it easier to buy but harder to sell. Thus the externalities associated with a change in production willingness can net out as positive or negative. A greater real money supply (willingness to produce held constant) implies a greater fraction of the population buying rather than selling or searching. This makes it easier to sell but harder to buy, again leaving no necessary sign on the net externality from increased real money. In this section we examine the determinants of the signs of these externalities. We do not analyze policies to bring about changes in c^* or m . With prices frozen, a change in c^* can be induced by subsidizing production, while a change in m is accomplished by giving money to some of the unemployed.

Denote by $W(e_0, m, c^*)$ the aggregate present discounted value of utility for an economy with an initial level of inventories e_0 and a constant level of real money and willingness to produce, c^* . W equals the present discounted value of consumption which has a flow utility level $yf(e, m)$ less the present discounted value of labor disutility, which has the flow value $a(n - e - m) \int_0^{c^*} cdG$. Thus we have

$$W(e_0, m, c^*) = \int_0^{\infty} e^{-rt} [yf(e, m) - a(n - e - m) \int_0^{c^*} cdG] dt \quad (19)$$

$$\text{s.t. } \dot{e} = aG(c^*)(n - e - m) - f(e, m)$$

$$e(0) = e_0$$

It is straightforward to calculate the derivatives of W with respect to m and c^* , evaluated at a value of e_0 where $\dot{e}$ is zero (the calculations are in Appendix B).

Differentiating, we have

$$\frac{\partial W}{\partial c^*} = \frac{aG'(n - e - m)}{r} \left(\frac{yf_e + a \int_0^{c^*} cdG}{r + f_e + aG} - c^* \right) \quad (20)$$

$$\frac{\partial W}{\partial m} = \left(\frac{f_m + aG}{r} \right) \left(\frac{yf_m + a \int_0^{c^*} cdG}{f_m + aG} - \frac{yf_e + a \int_0^{c^*} cdG}{r + f_e + aG} \right) \quad (21)$$

We want to examine these derivatives at an equilibrium willingness to produce - where (13) holds. Rewriting (13) as

$$((r+b)(r+s) + (r+b+s)aG)c^* = bsy + (r+b+s) a \int_0^{c^*} cdG \quad (22)$$

and substituting in (20) we have

$$\frac{\partial W}{\partial c^*} = \frac{aG'(n - e - m)}{r} \left(\frac{yf_e + a \int_0^{c^*} cdG}{r + f_e + aG} - \frac{bsy + (r+b+s) a \int_0^{c^*} cdG}{(r+b)(r+s) + (r+b+s)aG} \right) \quad (23)$$

Simplifying, we have

$$\text{sign } \frac{\partial W}{\partial c^*} = \text{sign } \left(f_e - \frac{bs}{r+b+s} \right) \quad (24)$$

provided we have an interior solution ($G' > 0$). To interpret this condition note that the individual goes through a two-step process to convert inventory into

utility. The arrival rates of opportunities for the steps are b and s . Thus someone who receives inventory costlessly whenever a sale and purchase is completed has expected utility* $bsy/r(r+b+s)$. Thus (24) represents the difference between the flow social marginal product of permanently higher inventory and the flow individual value of permanently higher inventory. Since there are no interactions in the production process, the costs drop out of the comparison.

Equation (24) holds at an equilibrium value of c^* and any value of m . Restricting our attention to equilibria satisfying the pricing condition, (24) becomes

$$\text{sign } \frac{\partial W}{\partial c^*} = \text{sign} \left(f_e - \frac{f}{2e} \right) \quad (25)$$

That is, if it were possible to permanently alter willingness to produce without altering the real money supply, it would be socially worthwhile to raise the willingness to produce if the marginal product of inventories in the trade process exceeded half the average product. It is interesting to note that $\frac{\partial W}{\partial c^*} > 0$ in the symmetric linear case, $f = k(e+m)$, since the pricing rule implies $e > m$. Without symmetry, $\partial W/\partial c^*$ could have either sign.

We turn now to a change in the real money supply. Rearranging terms in (21) we see that

$$\text{sign } \frac{\partial W}{\partial m} = \text{sign} \left(r(yf_m + a \int_0^{c^*} cdG) + (f_m - f_e) a \int_0^{c^*} (y - c)dG \right) \quad (26)$$

Let us consider the special case where $\frac{\partial W}{\partial c^*}$ is zero. Then (26) becomes

$$\text{sign } \frac{\partial W}{\partial m} = \text{sign} \left((y - c^*)f_m - a \int_0^{c^*} (c^* - c)dG \right) \quad (27)$$

There are two sources of surplus in this economy. One is from the greater utility of consumption than the marginally acceptable cost of production. The

* The value equations would be $rW_e = b(W_m - W_e)$ and $rW_m = s(y - W_m + W_e)$.

second is from finding opportunities that cost less than the marginally acceptable cost. Shifting people from searching to shopping speeds up the realization of the first gains, and decreases the tendency to pick up the second gain.

Considering equilibria satisfying the equal utility gain pricing rule, we note that if $\frac{\partial W}{\partial c^*} \leq 0$ then $\frac{\partial W}{\partial m} > 0$. From (26) $\partial W/\partial m$ is positive if $f_m \geq f_e$. The latter follows from $\partial W/\partial c^* \leq 0$ and nondecreasing returns since with $2f_e \leq f/e$, (2), and (16) we have

$$2f_m > f/m = f/e + r > f/e \geq 2f_e \quad (28)$$

Thus, in equilibrium, welfare is increasing in inventories or real money (or possibly both) the other held constant.

If the government can control production as well as the real money supply, optimization will imply an asymptotic steady state satisfying $\partial W/\partial m = 0$. The asymptotically optimal money supply must reflect the fact that more money implies more time spent shopping. Supplying more money in this model is not a way of making nonliquid wealth liquid. Rather, more real money is more wealth and so more shopping and less production.

8. Velocity of Money

The instantaneous income velocity of money is $f(e,m)/m$, an endogenous variable.* Across equilibria, equilibria those with higher e have higher m .

*To explore the determination of velocity more thoroughly, one should add search intensity variables to the transactions technology. In addition, credit and the use of money for transactions in assets would affect the measured income velocity. Further complications could be introduced by disaggregating commodities and the transactions technology.

With increasing returns, at least one of f/m or f/e must then be larger. By the pricing equation, (16), both f/m and f/e move in the same direction. Thus velocity is higher at the equilibrium with the greater production level.

If the transactions technology is homothetic one can also conclude that the pricing equation, (16), appears as in Figure 1, crossing rays through the origin from below. To see this, let us first consider constant returns to scale. Then, the pricing condition, (16), is a ray through the origin. With homotheticity and increasing returns, $r + f/e - f/m$ has at most one zero on any ray, being first positive and then negative. Since (16) is positively sloped and comes out of the e axis, it can only cut rays from below.

9. An Alternative Formulation

In the model above there are three mutually exclusive activities, buying, selling, and searching for production activities. Additional real money in the economy increases the number of buyers and so decreases the numbers engaged in either search or selling. This result follows because someone with money prefers to buy and consume before searching again rather than searching and selling followed by buying twice. The supply of real money has a second effect on the economy since a change in the number of shoppers changes the profitability of production and so the willingness to produce.

As an alternative formulation, assume that buying can take place simultaneously with selling or searching. (Think of couples.) Then individual workers divide their time between searching and selling. The division of time depends on the willingness to produce and the arrival rate of buyers. Now, willingness to produce varies with money available for purchase - those with more money have a lower cutoff cost for acceptance of a production opportunity. As a special case, if there is no willingness to produce when money holdings are sufficient for a purchase ($c^* < \underline{c}$), then this alternative formulation is equivalent to the previous formulation. Thus it is appropriate to say that additional real money has a wealth effect, decreasing willingness to produce. In addition, by making sales easier and purchases harder, additional real money has a second effect on willingness to produce.

10. Optimum Quantity of Money

The optimum quantity of money is normally discussed in two different contexts. One, focusing on the transactions role of money, often holds constant (or treats as controllable) private decisions. The other, typically focusing on capital accumulation, concentrates on the response of the private economy to changes in the supply of money or debt (e.g. in a growth model with overlapping generations or infinitely lived agents). We will look at both aspects. We proceed by contrasting the role of money in the two formulations presented above and then by comparing the analysis here to that of Grandmont and Younes (1973), which is an example of a discrete time model with a finance constraint.

In the alternative formulation given above, shopping is costless since it can be carried out at the same time as either searching or selling. The greater the money supply, the greater the fraction of the economy able to purchase and so the more efficient the transactions process (willingness to produce held constant). In the basic formulation, shopping is time consuming and there is an optimal fraction of the population engaged in shopping, and so, an implied optimal real money supply (willingness to produce held constant). The fact that paper money is costless to produce does not then imply that the economy should be saturated with money. In both formulations, willingness to produce varies with the money supply. In the absence of additional policy tools to control willingness to produce, the optimal money supply must reflect the effect of greater wealth on willingness to produce.

In the Grandmont-Younes paper, individuals are subjected to a transactions constraint as well as a budget constraint. Additional money eases the transaction constraint, achieving the decentralized full optimum in some cases. This is similar to the alternative formulation here where the transactions are costless (on the demand side). Since trips to the bank are a small fraction of individual shopping time, the basic formulation seems a more appealing framework for analyzing the transactions role of money (although both formulations are capable of exhibiting macro unemployment problems). However, one needs an alternative financial asset to adequately address this issue since the substitution of liquid for illiquid assets, wealth held constant, is likely to be different from the creation of additional (liquid) assets.

11. Sticky Prices

In a bargaining setting, it seems appropriate to describe prices as sticky if individuals who have met do not adjust prices to carry out mutually

advantageous trades.* In the model as formulated, there is no reason for sticky prices in this sense. One interesting project would be to reformulate the model so that there were reasons for sticky prices in this sense (such as insurance with limited observability, or long term contracts, or fairness constraints). Then, one could examine how sticky prices change the properties of equilibrium. An easier project would be to impose sticky prices on this economy to examine the induced changes in equilibrium properties.

This paper considers neither of these questions. Rather, in this setting of flexible prices, I want to note the appearance not of sticky prices, but of perverse price movements. If we compare the economy at times when it is in two different steady state equilibria, the economy with the greater production level has a lower price level. This can not be interpreted as saying that excessive prices are the cause of the low production level. If one froze prices in this economy at any desired level (and so froze the real money supply), one would still have multiple steady state equilibria, as shown in Appendix A. To analyze the problem of keeping the economy at the best equilibrium in the face of macro shocks, one needs a non-steady state model, which would go beyond the limits of this paper. In considering only steady states, the paper is limited to analysis of the local efficiency properties of equilibrium and can only point up the existence of these stabilization problems.

*An alternative approach to sticky prices would be that with a change in the economy prices no longer satisfy the rule for price determination, even though no mutually advantageous deals are missed.

12. Concluding Remarks

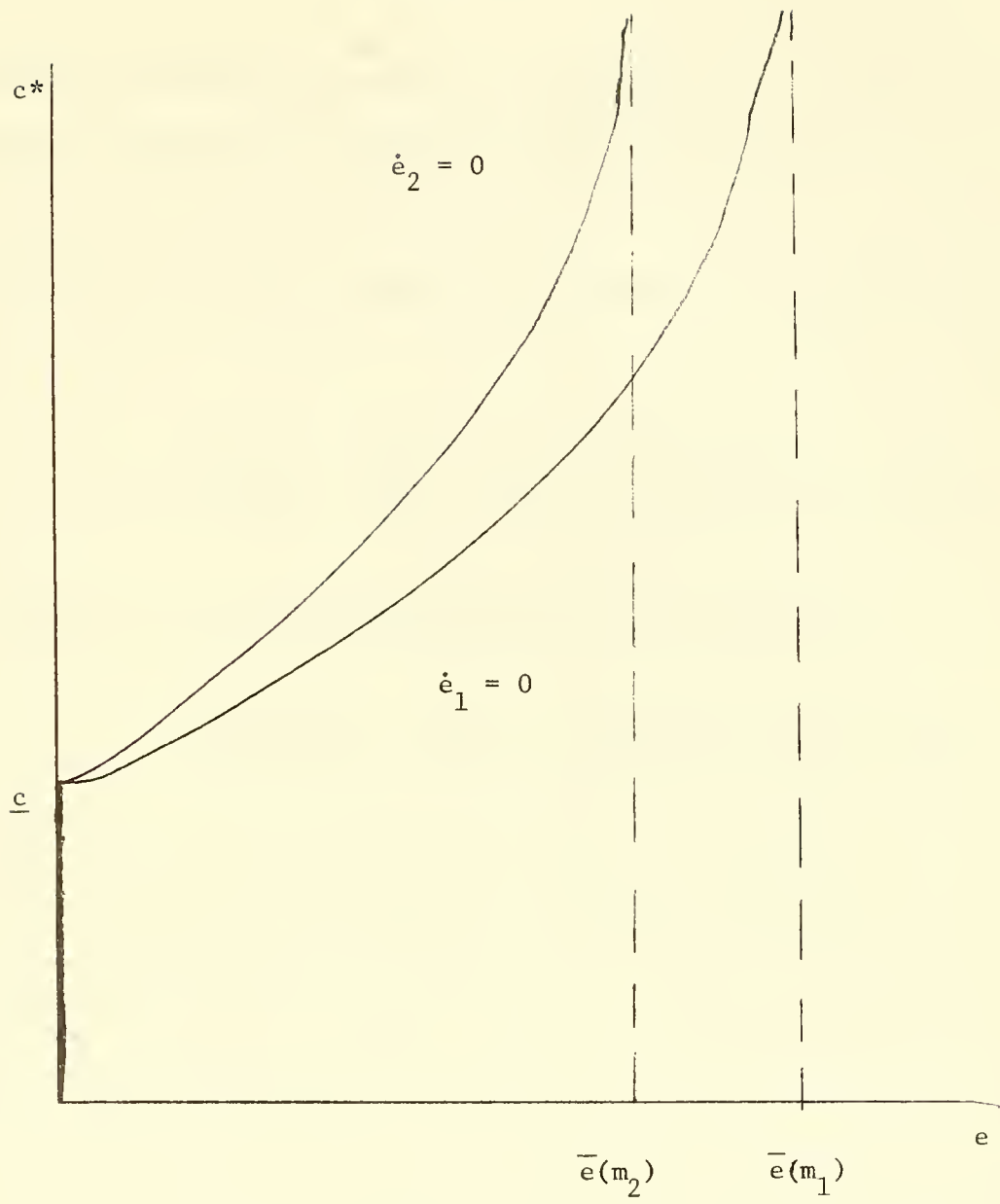
The Arrow-Debreu model plays a central role in the analysis of micro-economists. Models of particular phenomena (externalities, noncompetitive behavior, missing markets) are generally constructed preserving all other features of the competitive general equilibrium model. This has been an extraordinarily fruitful way of organizing and conducting research into the range of microeconomic questions. Micro based models addressing macro questions have generally (implicitly or explicitly) followed the same approach. Examples are Lucas (1972) where separate markets, with a random division of suppliers, preserve their competitive properties, and Lucas and Prescott (1974) where search in the labor market is combined with a competitive output market.

In the Arrow-Debreu model there is no explicit resource- or time-using trade coordination device. Thus, there is no natural transactions role for money. In addition, it is plausible that the difficulties of trade coordination are a major factor in the development of macro problems and in their response to government policies. If there is to be a firm micro based theory of money and macro problems, it seems likely that it must dispense with the fictitious Walrasian auctioneer. This paper is a start on this task - the construction of micro based general equilibrium models with a natural transactions role for money and the possibility of macro unemployment difficulties. By building on this start, it may be possible to achieve a greater understanding of modern economies and of the design of successful monetary and fiscal policies.

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Figure A1



Loci where $\dot{e} = 0$

Differentiating (A4) we have

$$\frac{\partial \bar{e}}{\partial m} = - \frac{a + f_m}{a + f_e} < 0 \quad (A5)$$

Thus, for $m_2 > m_1 > 0$ we have the curves shown in Figure A1 and labeled $\dot{e}_2=0$, $\dot{e}_1=0$. Since G need not have nice properties, the curve has no necessary concavity.

To analyze (A2), we must use (14) which states the derivatives of c^* with respect to b and s .

$$\begin{aligned} \frac{\partial c^*}{\partial e} &= \left(\frac{ry + a \int_0^{c^*} (c^* - c) dG}{(r+b)(r+s) + (r+b+s)aG} \right) \left(\frac{s}{r+b} \left(-\frac{ef_e - f}{e^2} \right) + \frac{b}{r+s} \left(\frac{f_e}{m} \right) \right) \\ &= \left(\frac{ry + a \int_0^{c^*} (c^* - c) dG}{(r+b)(r+s) + (r+b+s)aG} \right) \left(\frac{f}{em(r+b)(r+s)} \right) ((2r+b+s)f_e - (r+s)f/e) \end{aligned} \quad (A6)$$

$$\begin{aligned} \frac{\partial c^*}{\partial m} &= \left(\frac{ry + a \int_0^{c^*} (c^* - c) dG}{(r+b)(r+s) + (r+b+s)aG} \right) \left(\frac{s}{r+b} \left(\frac{f_m}{e} \right) + \frac{b}{r+s} \left(-\frac{mf_m - f}{m^2} \right) \right) \\ &= \left(\frac{ry + a \int_0^{c^*} (c^* - c) dG}{(r+b)(r+s) + (r+b+s)aG} \right) \left(\frac{f}{em(r+b)(r+s)} \right) ((2r+b+s)f_m - (r+b)f/m) \end{aligned} \quad (A7)$$

Substituting for b and s (from (4)) one has

$$e \frac{\partial c^*}{\partial e} + m \frac{\partial c^*}{\partial m} = \left(\frac{ry + a \int_0^{c^*} (c^* - c) dG}{(r+b)(r+s) + (r+b+s)aG} \right) \left(2r + \frac{f}{e} + \frac{f}{m} \right) (ef_e + mf_m - f) \left(\frac{f}{em(r+b)(r+s)} \right)$$

Thus, if there were locally constant returns, one of the derivatives (A6) or (A7) would be positive and the other negative. With the assumed increasing returns, both derivatives might be positive. Assuming a unique turning point of c^* in e , we have the shapes shown in Figure A2 where $c_1^*(e)$ is drawn for value m_1 with $m_1 < m_2$.

In Figure A3 we combine Figures A1 and A2 under the assumptions that the economy has equilibria other than the shut-down equilibrium and that these occur where c^* is rising in e and before the intersection of c_1^* and c_2^* . Thus there are at least three equilibria. There can be many more depending on the parameters. Equilibria at different m values are marked E_i , where $m_1 < m_2$. The picture can be quite different since the c^* curves can cross before some of the equilibria.

Assuming price controls and naive expectations it would be straightforward to analyze dynamics and so monetary policy. More interesting dynamic assumptions would be more difficult to analyze.

Figure A2

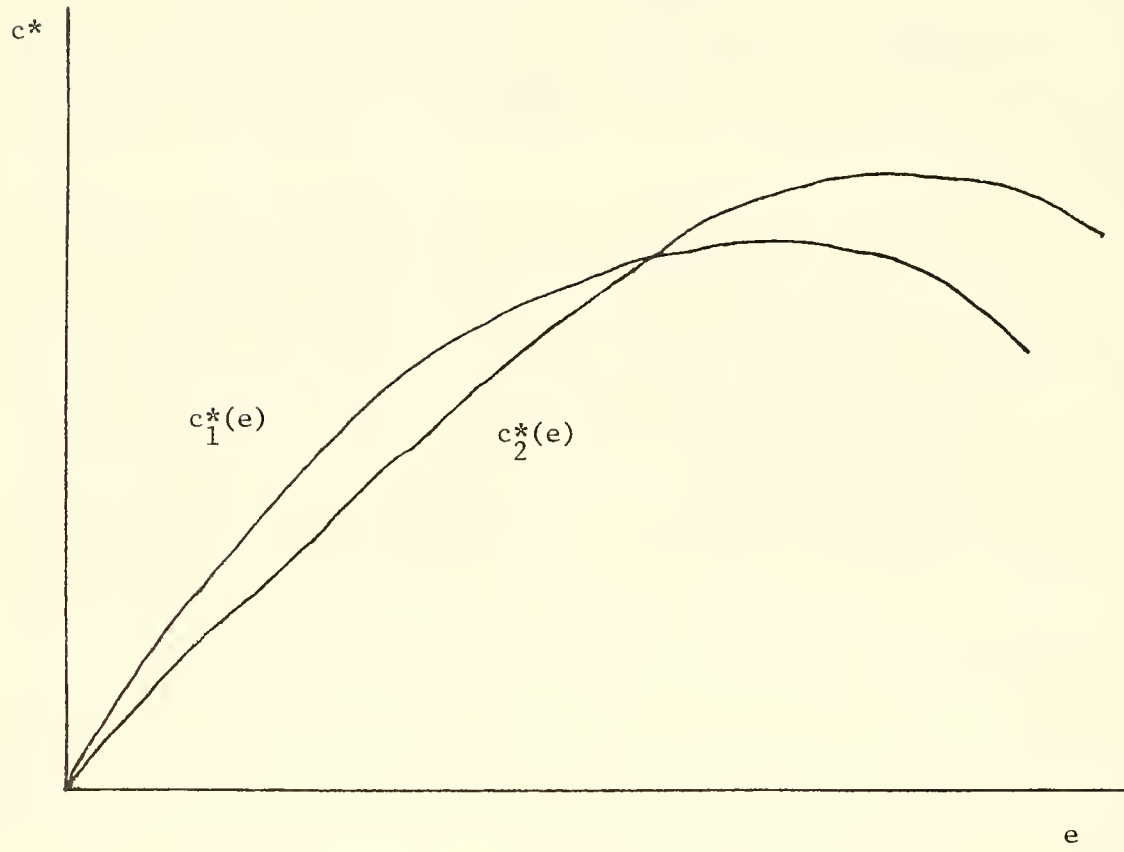
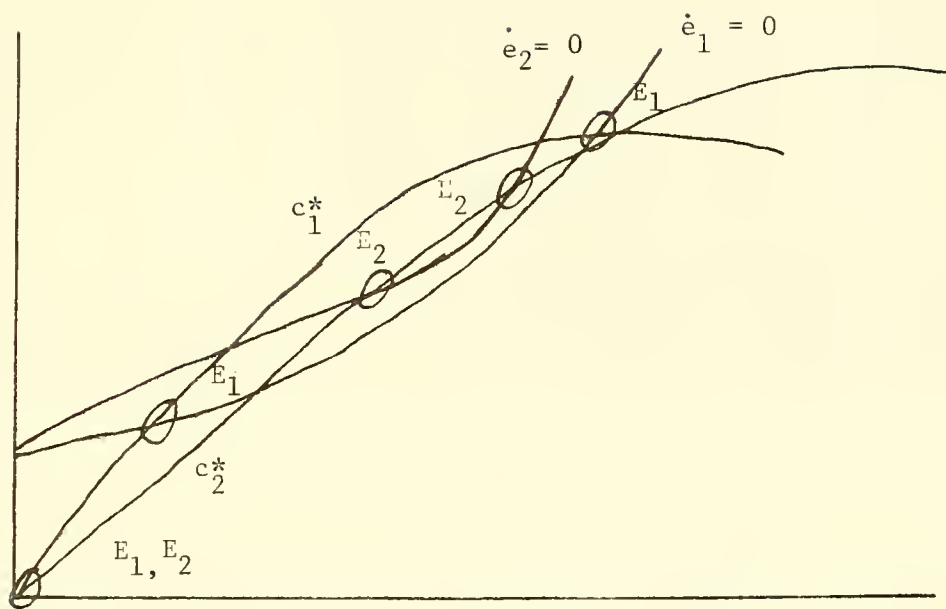


Figure A3



APPENDIX B

Derivation of Welfare Derivatives

We derive equations (20) and (21), using the method spelled out more fully in Diamond (1980). From (19) we have

$$rW = yf(e,m) - a(n-e-m) \int_0^{c^*} cdG + \frac{\partial W}{\partial e} \dot{e} \quad (B1)$$

Differentiating and evaluating at $\dot{e} = 0$, we have

$$r \frac{\partial W}{\partial e} = y \frac{\partial f}{\partial e} + a \int_0^{c^*} cdG - \frac{\partial W}{\partial e} (aG(c^*) + \frac{\partial f}{\partial e}) \quad (B2)$$

$$r \frac{\partial W}{\partial m} = y \frac{\partial f}{\partial m} + a \int_0^{c^*} cdG - \frac{\partial W}{\partial e} (aG(c^*) + \frac{\partial f}{\partial m}) \quad (B3)$$

$$r \frac{\partial W}{\partial c^*} = -a(n-e-m)c^*G' + \frac{\partial W}{\partial e} (aG'(n-e-m)) \quad (B4)$$

Solving (B2) for $\frac{\partial W}{\partial e}$ and substituting in (B3) and (B4) we have (21) and (20).

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